(1) Find the generators of the groups $\mathbb{Z}_6$, $\mathbb{Z}_8$, $\mathbb{Z}_12$ and $\mathbb{Z}_{60}$.

(2) Let $p$ be a prime number. Show that the trivial subgroups are the only subgroups of $\mathbb{Z}_p$. That is, show that if $0 \neq x \in \mathbb{Z}_p$, then $\mathbb{Z}_p = \langle x \rangle$.

(3) Let $p$ and $q$ be two prime numbers. How many generators does the group $\mathbb{Z}_{pq}$ have?
   (Hint: The order of $\mathbb{Z}_{pq}$ is $pq$. Not all of its elements are generators and you need to count them correctly. There are two cases: either $p = q$ or $p \neq p$.)

(4) Determine all subgroups of $\mathbb{Z}_{36}$ and draw the lattice diagram for this group.
   (Hint: The group $\mathbb{Z}_{36}$ contains a subgroup which is of the same type as the $\mathbb{Z}_{18}$. You can use the knowledge from the lecture notes about the subgroups of the group $\mathbb{Z}_{18}$ to determine its subgroup structure!)

(5) (a) Consider the cyclic group $\mathbb{Z}_{30}$. Find the number of elements of the subgroup generated by the element $25 \in \mathbb{Z}_{30}$.
   (b) Consider the cyclic group $\mathbb{Z}_{42}$. What is the order of the subgroup $\langle 30 \rangle \leq \mathbb{Z}_{42}$?
   (c) Consider the cyclic group $\mathbb{Z}_{42}$. What is the order of the element $25 \in \mathbb{Z}_{42}$?

(6*) The monster group $M$ (also known as the Fischer-Griess Monster or The Friendly Giant) is the highest order sporadic group.\footnote{see for example http://en.wikipedia.org/wiki/Monster_group} The order of $M$ is
   \[ |M| = 8080174247945128758864599049617107570057543680000000000 \]
   \[ \approx 8 \cdot 10^{53}. \]

It is in no way a cyclic group. However, in this exercise we will not study the Friendly Giant but rather we consider the group

\[ G := \mathbb{Z}_{8080174247945128758864599049617107570057543680000000000}, \]

that is the cyclic group which has the same order as the Friendly Giant.

(a) Determine the order of the element $323 \in G$
(b) Determine the subgroup $\langle 37 \rangle$. 

\[ G := \mathbb{Z}_{8080174247945128758864599049617107570057543680000000000}. \]