Some additional exercises for practicing the methods for solving systems of linear equations as introduced in the first chapter of the lecture notes. We will solve and discuss them in the exercise session on Friday, January 26th. (Note that these exercises do not count for the grade.)

1. Find all solutions to the following homogeneous system of linear equations:
   \[ 
   \begin{align*}
   3x_1 + 9x_2 - x_3 - 4x_4 - 4x_5 &= 0 \\
   9x_1 + 19x_2 + 17x_3 - 12x_4 &= 0 \\
   2x_2 - 5x_3 - 3x_5 &= 0 \\
   -3x_1 - 11x_2 + 6x_3 + 4x_4 + 7x_5 &= 0 \\
   3x_1 - 7x_2 + 4x_3 - 4x_4 - x_5 &= 0 
   \end{align*}
   \]

2. Complete
   \[ x := \begin{pmatrix} 2 & x_2 & x_3 \\ x_4 & x_5 & 1 \\ x_7 & 0 & x_9 \end{pmatrix} \]
   to a magic square if possible.

3. Find all solutions to the following homogeneous system of linear equations:
   \[ 
   \begin{align*}
   x_1 + 2x_2 + x_3 + x_4 + x_5 &= 0 \\
   -x_1 - 2x_2 - 2x_3 + 2x_4 + x_5 &= 0 \\
   2x_1 - 4x_2 + 3x_3 - x_4 &= 0 \\
   x_1 + 2x_2 + 2x_3 - 2x_4 - x_5 &= 0 
   \end{align*}
   \]

4. (a) Find all solutions to the following nonhomogeneous system of linear equations:
   \[ 
   \begin{align*}
   x_1 + x_2 &= b_1 \\
   3x_1 - 2x_2 &= b_2 
   \end{align*}
   \]
   where \( b_1 \) and \( b_2 \) are some numbers.

   (b) Find all solutions to the following homogeneous system of linear equations:
   \[ 
   \begin{align*}
   x_1 + x_2 + x_3 &= 0 \\
   3x_1 - 2x_2 + 5x_3 &= 0 \\
   9x_1 + 4x_2 + 25x_3 &= 0 
   \end{align*}
   \]

   (c) Find all solutions to the following nonhomogeneous system of linear equations:
   \[ 
   \begin{align*}
   x_1 + x_2 + x_3 + x_4 &= 0 \\
   -x_1 - 3x_2 + x_3 + 2x_4 &= 2 \\
   x_1 + 9x_2 + x_3 + 4x_4 &= 1 \\
   -x_1 - 27x_2 + x_3 + 8x_4 &= -1 
   \end{align*}
   \]

5. (a) Find all solutions to the following homogeneous system of linear equations:
   \[ 
   \begin{align*}
   13x_1 + 13x_2 + 5x_3 - 9x_4 &= 0 \\
   x_1 - x_3 + 3x_4 &= 0 \\
   5x_1 + 3x_2 + 2x_4 &= 0 \\
   7x_2 + 8x_3 - 4x_4 &= 0 
   \end{align*}
   \]
(b) Find all solutions to the following nonhomogeneous system of linear equations:
\[
\begin{align*}
13x_1 + 13x_2 + 5x_3 - 9x_4 &= 0 \\
x_1 - x_3 + 3x_4 &= 1 \\
5x_1 + 3x_2 + 2x_4 &= 2 \\
7x_2 + 8x_3 - 4x_4 &= -7
\end{align*}
\]

(c) Find all solutions to the following nonhomogeneous system of linear equations:
\[
\begin{align*}
13x_1 + 13x_2 + 5x_3 - 9x_4 &= 1 \\
x_1 - x_3 + 3x_4 &= 0 \\
5x_1 + 3x_2 + 2x_4 &= 0 \\
7x_2 + 8x_3 - 4x_4 &= 0
\end{align*}
\]